Une approche spectrale pour l'analyse de corrélation sur la consommation

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Introduction

Side Channel Attack

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A cryptographic device (smart card, FPGA device...) performs operations on secret data that are hold inside the circuit of the devices:

- running the cryptographic algorithm $\Longrightarrow$ physical emanations or leakage due to functions that operate on secret data (ex. non-linear function)
- exploiting physical leakage $\Longrightarrow$ physical attacks that reveal the secret data

This kind of attack is called Side Channel Attack or SCA for short.

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Correlation Power Analysis (CPA)

Side Channel Attack

- Simple Power Analysis (SPA) [Koc96]
- Differential Power Analysis [PJB99]
- Correlation Power Analysis (CPA) [BCO04]

Correlation Power Analysis (CPA)

Side Channel Attack

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Test Bench:

- smart card (containing the cryptographic algorithm to attack)
- oscilloscope (to perform measurement when the smart card is working)
- Personal Computer (to drive the smart card and the oscilloscope).

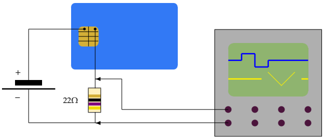

Figure : Test bench.

Correlation Power Analysis

A CPA establishes correlation between :

- real power consumption values obtained by measuring the power consumption when running the smart card

  and

- hypothetical power consumption values obtained from a leakage model.

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Function on binary words

$\varphi : \{0,1\}^n \mapsto \mathbb{R}$: Function on binary words. $\Phi = \{\varphi : \{0,1\}^n \to \mathbb{R}\}$

Scalar product

$\varphi, \psi \in \Phi$:

$$\langle \varphi, \psi \rangle = \sum_{x \in \{0,1\}^n} \varphi(x)\psi(x).$$

Norm

$$\|\varphi\| = \sqrt{\langle \varphi, \varphi \rangle} = \sqrt{\sum_{x \in \{0,1\}^n} \varphi(x)^2}$$

The norm of $\varphi \in \Phi$ is called energy of φ.

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Fourier Transform

$$\begin{array}{rccl} \widehat{\varphi}(u) : & \{0,1\}^n & \mapsto & \mathbb{R} \\ & u & \mapsto & \widehat{\varphi}(u) = \dfrac{1}{\sqrt{2^n}} \displaystyle\sum_{x \in \{0,1\}^n} \varphi(x)(-1)^{u \cdot x} \end{array}$$

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Isometry of Fourier Transform

$$\langle \varphi, \psi \rangle = \langle \widehat{\varphi}, \widehat{\psi} \rangle.$$

Energy conservation law

$$\|\varphi\| = \|\widehat{\varphi}\|$$

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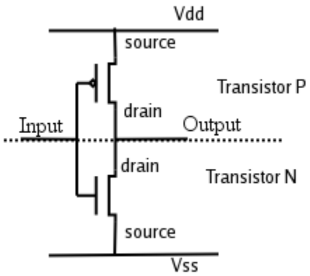
Physical principles

Circuit CMOS (Complemantary Metal-Oxyde Semiconductor):



Hypothesis

The power consumption of a CMOS circuit is proportional to the number of logic gates that switch.

Target of the attack

The target of the CPA attack is an S-Box function implemented as Look Up Table (LUT) in the circuitry of the device.

S-Boxes performed essentially non-linear operations of the encryption algorithm.

S-Box

$$f : \{0,1\}^n \mapsto \{0,1\}^m$$

it computes a quantity (within the cryptographic device)

$$y = f(x + k^\star)$$

where x is a known value (by the adversary) and $k^\star$ is an unknown value (by the adversary)

$\Longrightarrow$ aim of the CPA attack on the S-Box: recover the value $k^\star$.

Example $n = m = 4$, $f : \{0,1\}^4 \mapsto \{0,1\}^4$

0	1	9	14	13	11	7	6	15	2	12	5	10	4	3	8

Leakage model and estimation of the power consumption φ

Leakage model (Hamming weight)

$$\varphi(x) = \mathrm{WH}(f(x+k^\star)) + \varepsilon(x) + C$$

$$\varphi(x) = \sum_{i=1}^{m} f_i(x+k^\star) + \varepsilon(x) + C \qquad (1)$$

- $\sum_{i=1}^{m} f_i(x+k^\star)$ is the Hamming weight of f
- $\varepsilon(x)$ is a random noise that depends on x
- C is a constant related to the power consumption of the smart card that does not depend on the value x.

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The adversary needs to guess the value of $k^\star$ and then assume a value k that is not equal to $k^\star$.
For this value k (with $g(x+k) = \sum_{i=1}^{m} f_i(x+k)$:= power consumption model):

$$\varepsilon_k(x) = \varphi(x) - g(x+k) - C. \qquad (2)$$

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likelihood value of $k^\star$

The value of $k^\star$ is the value of k for which the energy of the noise ε_k is minimal:

$$k = k^\star \Leftrightarrow \|\varepsilon_k(x)\| \text{ minimal for } k$$

Steps of the CPA attack

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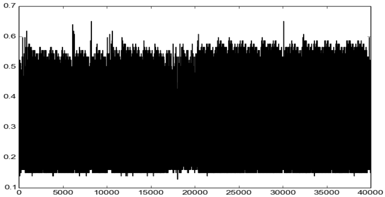

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3. analyse the traces by applying Fourier transform to find the probable value of $k^\star$

Estimation of the likelihood value of $k^\star$

Energy conservation law $\Longrightarrow$ $\|\varepsilon_k\| = \|\widehat{\varepsilon_k}\| = \sqrt{\sum_{u\in\{0,1\}^n} \widehat{\varepsilon_k}(u)^2} = E(k)$

find $k^\star$ $\Longrightarrow$ find $\arg\min_k \sum_{u\in\{0,1\}^n} \widehat{\varepsilon_k}(u)^2 = \arg\min_k E(k)^2$

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$$\text{Equation (2)} \Rightarrow \qquad \widehat{\varepsilon_k}(u) = \widehat{\varphi_t}(u) - (-1)^{u\cdot k}\widehat{g}(u) - C\sqrt{2^n}\delta_0(u). \tag{3}$$

The constant value C can be discarded by considering the functions:

$$\widehat{\varphi_t^\star}(u) = \begin{cases} \widehat{\varphi_t}(u) & \text{if } u \neq 0 \\ 0 & \text{else} \end{cases} \text{ and } \widehat{g}^\star(u) = \begin{cases} \widehat{g}(u) & \text{if } u \neq 0 \\ 0 & \text{else} \end{cases} \tag{4}$$

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$$E(k)^2 = \sum_{u\in\{0,1\}^n} \widehat{\varphi_t^\star}(u)^2 + \sum_{u\in\{0,1\}^n} \widehat{g}^\star(u)^2 - 2\sum_{u\in\{0,1\}^n} \widehat{\varphi_t^\star}(u)\widehat{g}^\star(u)(-1)^{u\cdot k} \tag{5}$$

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$\Longrightarrow$ **minimize** $E(k)^2$ amounts to **maximize** $F_t(k) = \sum_{u\in\{0,1\}^n} \widehat{\varphi_t^\star}(u)\widehat{g}^\star(u)(-1)^{u\cdot k}$

The theoretical value of $k^\star$ is then given by:

$$\arg\max_k F_t(k) \tag{6}$$

Estimation of the reliability of the value k that is found

The reliability of the value k that is found can be estimated by:

$$r_t(k) = \frac{F_t(k)}{\|\widehat{\varphi}_t^\star\| \cdot \|\widehat{g}^\star\|} = \frac{\langle \widehat{\varphi}_t^\star, \widehat{g}_k{}^\star \rangle}{\|\widehat{\varphi}_t^\star\| \cdot \|\widehat{g}_k{}^\star\|}$$

This coefficient is the Pearson correlation coefficient of φ_t (the real consumption value) and g (the hypothetical consumption value):

$$r_t(k) = \rho(\varphi_t, g) \qquad \text{with} \qquad \rho(\varphi, \psi) = \frac{\langle \varphi - m_\varphi, \psi - m_\psi \rangle}{\|\varphi - m_\varphi\| \cdot \|\psi - m_\psi\|}$$

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Pearson correlation coefficient

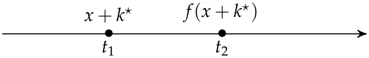
Let two random variables X and Y:

$$\rho(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{\sum_{i=1}^n (X_i - \bar{X}) \cdot (Y_i - \bar{X})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2 \cdot \sum_{i=1}^n (Y_i - \bar{Y})^2}}$$

- $\rho(X,Y) \in [-1,1]$
- $|\rho| = 1 \Longrightarrow$ high correlation
- $\rho = 0 \Longrightarrow$ low correlation

the correlation coefficient measures statistical relationship (”proportionnality”) between two random variables.

Multidimensional attack



multidimensional attack $\Longrightarrow$ attack that takes into account consumption at several instants

time		computed value	consumption	leakage model	error
t_1	$\rightarrow$	$f^1(x+k^\star)$	φ^1	g^1	E^1
t_2	$\rightarrow$	$f^2(x+k^\star)$	φ^2	g^2	E^2

Multidimensional attack

$$\xrightarrow[\quad t_1 \quad\quad\quad t_2 \quad]{x+k^\star \quad\quad f(x+k^\star)}$$

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The most likely value of $k^\star$: minimize $\|\overrightarrow{E}\| = \|(E^1, E^2)\|$

$\Longleftrightarrow$ maximize $F(k) = \sum_{u \neq 0} \left(\widehat{\varphi^1}(u)\widehat{g^1}(u) + \widehat{\varphi^2}(u)\widehat{g^2}(u)\right)(-1)^{u \cdot k}$

$$\Phi_2 : \{\overrightarrow{\varphi} : \{0,1\}^n \to \mathbb{R}^2\}$$

$$\forall \overrightarrow{\varphi} = (\varphi^1, \varphi^2) \in \Phi_2$$
$$\forall \overrightarrow{\psi} = (\psi^1, \psi^2) \in \Phi_2$$

Euclidean scalar product in Φ_2

$$\langle \overrightarrow{\varphi}, \overrightarrow{\psi} \rangle = \langle \varphi^1, \psi^1 \rangle + \langle \varphi^2, \psi^2 \rangle$$

Euclidean norm in Φ_2

$$\|\overrightarrow{\varphi}\|^2 = \|\varphi^1\|^2 + \|\varphi^2\|^2$$

reliability coefficient $\Longrightarrow r_t(k) = \dfrac{\langle (\widehat{\varphi_t^1}^\star, \widehat{\varphi_t^2}^\star), (\widehat{g_k^1}^\star, \widehat{g_k^2}^\star) \rangle}{\|(\widehat{\varphi_t^1}^\star, \widehat{\varphi_t^2}^\star)\| \cdot \|(\widehat{g^1}^\star, \widehat{g^2}^\star)\|}$,

Experimental Results

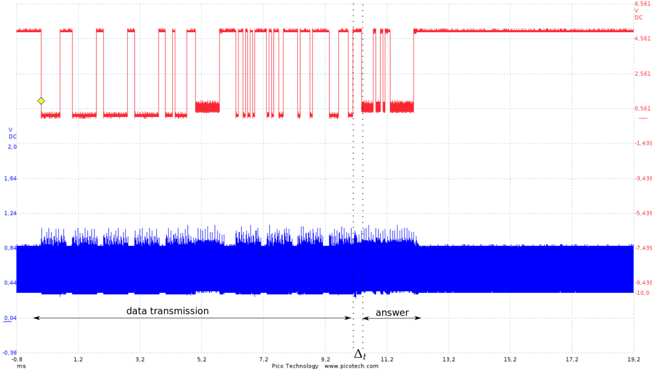
AES S-box (size of the S-box $n = 8$)

compute sequentially 4 times the S-box with different 8-bit keys k_1, k_2, k_3, k_4

record 256 traces of 40.000 samples

Experimental Results

power consumption and et I/O signal when running the encryption algorithm:



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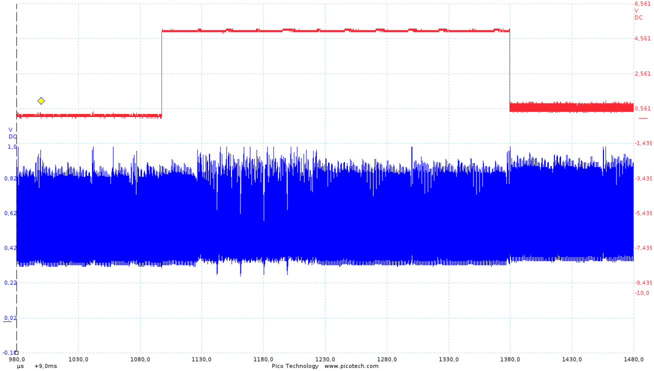
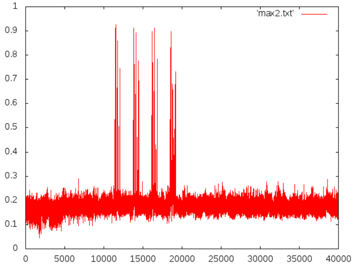
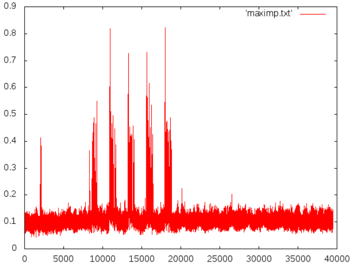


Experimental Results

Correlation between the real power consumption φ_t and the hypothetical power consumption g



One dimensional attack



Multidimensional attack

Conclusion

- we performed a CPA attack based on a Fourier Transform
- the approach allows multidimensional attack
- improvement: reduce the number of traces required to recover the right key

Thank you for your attention

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